Huygens' law of strange refraction could not be divorced from its wave-theoretical derivation. In the context of late 17th-century mathematics this is certainly true. However, would it not have been clear to Huygens that his law was still fully quantitative and that he just did not have the right techniques to be able to separate the enunciation of the law from the wave theory? Clearly this construction is very different from, say, the essentially qualitative Cartesian models of physics. Once the right tools became available in the early 1800s, Etienne Malus had little problem in separating the law from the wave theory. In short, I still do not really understand why Huygens felt that his wave-theoretic approach provided an explanation of the nature of light rather than an alternative description of its behavior. If Huygens really felt that way, it seems to me there is a certain irony in the fact that his *Traité* was bundled with a treatise in which Huygens accuses Newton of ignoring any discussion of the cause of gravity in the *Principia*. It would appear that there is no fundamental difference between Huygens' use of the concept of waves and Newton's introduction of the concept of force. Both are perfect illustrations of (E.J.) Dijksterhuis' "mechanization of the world picture." The only difference is that Newton was willing to ignore the issue of the ontological status of the concept of force, whereas Huygens could only conceive of waves as real, tangible entities. But then, some might argue, it is exactly these kinds of differences that made Newton a giant and Huygens merely great.

While I have little to comment on the actual contents of the book, I have a major quibble with the presentation of the material and I am not sure whether the author or the publisher should be blamed. The book was clearly published from a camera-ready manuscript and the text is riddled with typos, grammatical mistakes, and unusual (sometimes unfortunate) expressions—with most of the latter clearly recognizable as "Dutchisms." Overall, the author's English is perfectly understandable, but the book really could have used an editor. For what Kluwer charges for its publications ($109 for this one), you would think that the company could afford to hire one. Any reasonably well-educated native English speaker could have easily spared the author the embarrassment of publishing a book that occasionally reads like a policy memo put out by the European Union!

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The Mathematical Works of Bernard Bolzano
By Steve Russ. Oxford (Oxford University Press). 2004. ISBN: 0-19-853930-4. xxx + 698 pp. $229.50

Bernard Bolzano (1781–1848) is arguably the most important philosopher of mathematics to have had almost no influence in his own lifetime. He was prevented by the Bohemian authorities from publishing within the Hapsburg Empire between 1821 and 1835 on the grounds that his views were too liberal. The articles of his that were published before the ban were for the most part obscure even when they addressed questions of self-evident importance, and the later works, often unpublished in his lifetime, are often more philosophical and in their own ways obscure. This double obscurity was briefly lifted by some German writers in the middle of the 19th century, and he later became an important figure for the group around Brentano, but by then his strictly mathematical achievements had become mainstream through the work of others, and he seemed set to become a major footnote. His philosophy of mathematics was acute, however, and it is the rare combination of philosophical sharpness and genuine mathematics that, however precariously, contributes to the growing interest in, and appreciation of, his work. Husserl is quoted here on p. 1 declaring him one of the greatest logicians of all time, and he is almost the hero of Coffa's *The Semantic Tradition from Kant to Carnap* [Coffa, 1991].

English readers have been particularly poorly served by Bolzano scholarship, which now plans to produce an intimidating *Bernard Bolzano Gesamtausgabe* of 120 volumes in German; those already published are surely not well known. One or two of Bolzano's books have long been translated, for example the *Paradoxes of the Infinite* (PU),[2] and *Historia Mathematica* some years ago carried an early version of the RZ that now graces this volume [Russ, 1980]. Ewald's fine source book [Ewald, 1996] carries a translation of RB, BD, and BG; revised versions of these works appear here. Russ has also provided translations of BL and DP, as well as of a generous extract from RZ, F, and some of Bolzano's revisions to that work, F+. This selection includes some important work not appreciated in any language before the 20th century, and F contains Bolzano's example of a function that oscillates in every interval and is everywhere continuous and nowhere differentiable. The volume ends with a new translation of PU. We therefore have been presented with two essays on geometry and foundations, Bolzano's proofs of the binomial theorem and the fundamental theorem of algebra, some early investigations into the concept of the integral, and four later investigations into the concept of real numbers, function, and the infinite.

Russ introduces the book with a number of thoughtful remarks of many kinds. Some reflect his deep involvement with the process of making translations, where one feels that he might well agree with the American poet Lawrence Ferlinghetti that translations, unlike poems, are never finished, merely abandoned. The sensitivity Russ has brought to the task is reflected in his appreciation of the life of language, and it shows up in the clarity of the translations, which read extremely well.

He is also shrewd in his introductions, both to the book as a whole and to the individual works. We learn, for example, that whereas Bolzano was hostile to the idea of the infinite and its use in mathematics as a young man, in his later work he came round to its advantages and developed ideas about how it can validly be used. In his proof of the binomial theorem in BL, Bolzano rejected the infinitely small as a self-contradictory idea and founded this part of the study of the calculus on an idea of limits (not so unusual by then), and he would not countenance infinite series, only arbitrarily long finite parts of them. By the 1840s he was happy enough with the infinite to put it into the title of a book, the PU, and to admit at once that it is precisely in mathematics that people speak most frequently of the infinite. The implication is that they are not foolish to do so.

One of the mathematical topics Bolzano is rightly remembered for is his refusal to admit geometric considerations into the proof of theorems in analysis, most notably the fundamental theorem of algebra. This is evident both in the title of RB—*A purely analytic proof, etc.*—and in the methods Bolzano used; it becomes clear that one of the things Bolzano meant by analytic was nongeometric. Purity of method, as present-day philosophers of mathematics call the insistence on the use of only the most appropriate methods, was a deep concern of Bolzano, but he seems to have felt that it would earn him few readers and he largely gave up the activity for a number of years. Russ argues that this concern has been misunderstood by historians of mathematics, who have been too eager to see him as a precursor of those who rigorized the calculus by means of a sophisticated limit concept and an emphasis on the arithmetic character of the key definitions. Rather, he suggests, what Bolzano offered in his proof of the binomial theorem may not have been the modern limit concept but a way of dealing with finite variable quantities. On the other hand, the definition in RB of continuity of a function at a point seems to be the modern one, and therefore clearer than Cauchy's definition a little later, which only considered continuity on an interval.

Bolzano certainly claimed that if a sequence of quantities, u_n say, has the property that the absolute value of the difference $u_n - u_{n+r}$ can be made arbitrarily small for all values of r by taking n large enough, then the sequence converges to a limit. (Such sequences are called Cauchy sequences today.) A commonly held view is that his argument is flawed because Bolzano lacked a proper theory of the real numbers, but Russ suggests that the idea that sequences with this "bunching up" property are defining a number is a profound insight into the nature of number that Bolzano was perhaps the first to express.

A laudable feature of this book is that it is both up-to-date with, and fair to, current Bolzano scholarship. For example, the extract from RZ is on the vexed topic of measurable numbers. It grows out of Bolzano's earlier work on the fundamental theorem of algebra, as it must if that theorem is taken to require a proper theory of the real numbers before it can be rigorously proved. Here Russ defers to other scholars for a fuller analysis of this mixed achievement. Generally, it is his technique to hold open the possibility that Bolzano's work may admit unexpected interpretations rather than fit tidily into modern ones, and he gives several examples where older interpretations have had to be

[2] Bolzano's works translated in this volume (with Russ's abbreviations for them) are: BG: *Betrachtungen über einige Gegenstände der Elementargeometrie*, BD: *Beyträge zu einer begründeteren Darstellung der Mathematik*, BL: *Der binomische Lehrsatz, etc.*, RB: *Rein analytischer Beweis, etc.*, DP: *Die drey Probleme der Rectification, etc.*, RZ: *Reine Zahlenlehre*, F: *Functionenlehre*, F+: *Verbesserungen und Zusätze zur Functionenlehre*, and PU: *Paradoxien der Unendlichen*.

reexamined in the light of new evidence. He is also willing to allow that even when Bolzano is profound he may not have been saying what later mathematicians came to say.

The book is exemplary in a number of ways. One is that Russ has been at pains to say the minimum necessary to introduce Bolzano, and as a result we have a very generous amount of material here. His comments are judicious and informative; one might have asked for more. Another is that the selection not only covers the better known early works that would have to be in any one-volume translation, but brings up later works that show how Bolzano scholarship continues to advance. As already noted, the translations read fluently while remaining faithful to Bolzano's not always optimal choice of notation. Lastly, Oxford University Press is to be congratulated on producing a handsome book that matches the significance of its subject.

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Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability
By Peter Pesic. Cambridge, MA/London (MIT Press). 2003. ISBN 0-262-16216-4, 213 pp. $24.95

In 1824, Niels Henrik Abel published the first complete proof of the unsolvability of the quintic in radicals. Although the problem of finding solutions for the quintic had been weighing on the minds of great mathematicians for centuries, the proof received little attention at the time of its publication; even Gauss could not be bothered to read it. It was not until after Abel's death that the proof began to receive the attention it deserved.

It is easy to forget that each mathematical concept or piece of notation we employ has a rich history. In *Abel's Proof* by Peter Pesic, we are reminded that the mathematical tools we apply without question were once new and strange. Pesic's book is not just an explanation of a 180-year-old proof, it is a story about the acceptance of new mathematical ideas, ideas that may fly in the face of beliefs held for hundreds of years.

The atmosphere of controversy that surrounded the question of the solvability of the quintic in radicals is felt from the first chapter of Pesic's book. Although Pesic waits until much later to discuss Abel's proof, in the first chapter he provides the reader with a glimpse of the mathematical turmoil to come with his account of the "scandal of the irrational." He takes us back to ancient Greece and the myths that surround the Pythagorean brotherhood, including that the arrival of the irrational caused such upset that it may have resulted in the drowning of its discoverer.

Next, Pesic describes the gradual appearance of algebra as we now know it. We see that although we are still solving some of the same problems that were studied by the Babylonians (for example, quadratic equations), the method we use to formulate these problems and provide solutions has changed drastically. Interestingly, the general solutions for quadratic, cubic, and quartic equations all preceded the introduction of an algebraic notation for variables and coefficients by François Viète in the 16th century.

The solution of these equations was followed by another tumultuous period for new mathematical ideas. It was not just irrational numbers that had trouble finding a way into mathematical theory—negative and imaginary numbers